ANALYSIS OF THE BRANCHING OF A SPIRAL FLOW BETWEEN COAXIAL CYLINDERS

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Investigation in the linear approximation of the stability of the spiral flow of a viscous incompressible fluid between two coaxial cylinders due to an axial pressure gradient and rotation of the inner cylinder shows that several flow modes are observed, depending on the ratio of the governing parameters Re and T [1]. The general stability pattern is qualitatively confirmed by experiment [2].

The nature of the stability loss of the limiting cases of a spiral flow is diverse. In one case, in which the axial pressure gradient is absent, a circular Couette flow loses stability at a specified rotational velocity of the inner cylinder in the case of a small supercriticality, and a secondary steady-state flow — stable Taylor vortices — is weakly excited [3]. In the other limiting case, in which the inner cylinder is at rest and the axial pressure gradient is different from zero, the nature of the stability loss of the basic flow depends significantly on the wave number α of the perturbations [4].

For α close to the critical value α_*, as well as for all α on the upper branch of the neutral curve in the case of a small subcriticality, there exists a self-oscillating unstable mode which blends with a Poiseuille flow at a critical value of the Reynolds number. No self-oscillations exist in the vicinity of the Poiseuille flow in the case of a small supercriticality.

A stable self-oscillating mode arises on the lower branch of the neutral curve starting from some α when Re passes through the critical value.

An investigation of the nature of the stability loss of a spiral flow over a wide range of variation of Reynolds and Taylor numbers and width of the gap between cylinders is of interest.

The width of the gap between cylinders is adopted as the scale of length, and ξ is the dimensionless radius of the inner cylinder. The value of the axial component averaged over the transverse cross section of the channel is taken as the scale of velocity.

The dimensionless value of the azimuthal velocity on the surface of the inner cylinder v characterizes the kinematics of the basic flow. The Reynolds number is formulated from the selected length and velocity scales, and the Taylor number, expressed in terms of the selected dimensionless parameters, has the form $\sqrt{T} = 4\beta\,\mathrm{Re}$, where $\beta = 0.25\sqrt{2/(1 + 2\xi)}\cdot v$ is a dimensionless parameter which is independent of the viscosity.

Following [4], let us seek the self-oscillating solution in the form

$$v_r' = (1/\mathrm{Re})\, w_r(r, \varphi, z),\ v_\varphi' = (1/\mathrm{Re})\, w_\varphi(r, \varphi, z) + v_0(r),$$
$$v_z' = (1/\mathrm{Re})\, w_z(r, \varphi, z) + u_0(r), \quad p' = (1/\mathrm{Re})\, p(r, \varphi, z) + p_0(r),$$

where $z = z' - ct$; $v_0(r)$, $u_0(r)$, and $p_0(r)$ is the solution of the unperturbed steady-state problem.

The problem of calculating in dimensionless variables the wave perturbations of a spiral flow $\mathbf{v}(0, v_0, u_0)$ with velocity components

$$u_0 = Ar^2 - B\ln r + C,\ v_0 = E/r - Dr$$

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is a nonlinear problem for the eigenvalues of the phase velocity c of the waves

$$\frac{\partial r w_r}{\partial r} + \frac{\partial r w_z}{\partial z} + \frac{\partial w_\varphi}{\partial \varphi} = 0, \tag{1}$$

$$a\frac{\partial w_r}{\partial z} + w_r\frac{\partial w_r}{\partial r} + \frac{\mathrm{Re}\, v_0 + w_\varphi}{r}\frac{\partial w_r}{\partial \varphi} - \frac{2\mathrm{Re}\, v_0 + w_\varphi}{r} w_\varphi = -\mathrm{Re}\frac{\partial p}{\partial r} + $$
$$+ \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r w_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w_r}{\partial \varphi^2} + \frac{\partial^2 w_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial w_\varphi}{\partial \varphi},$$

$$a\frac{\partial w_\varphi}{\partial z} + w_r\frac{\partial w_\varphi}{\partial r} + \frac{\mathrm{Re}\, v_0 + w_\varphi}{r}\frac{\partial w_\varphi}{\partial \varphi} + \left[\mathrm{Re}\left(\frac{v_0}{r} + v_0'\right) + \frac{w_\varphi}{r}\right] w_r = -\frac{\mathrm{Re}}{r}\frac{\partial p}{\partial \varphi} + $$
$$+ \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w_\varphi r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w_\varphi}{\partial \varphi^2} + \frac{\partial^2 w_\varphi}{\partial z^2} + \frac{2}{r^2}\frac{\partial w_r}{\partial \varphi},$$

$$a\frac{\partial w_z}{\partial z} + w_r\frac{\partial w_z}{\partial r} + \frac{\mathrm{Re}\, v_0 + w_\varphi}{r}\frac{\partial w_z}{\partial \varphi} + \mathrm{Re}\, u_0' w_r = -\mathrm{Re}\frac{\partial p}{\partial z} + \frac{\partial^2 w_z}{\partial r^2} + \frac{1}{r}\frac{\partial w_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_z}{\partial \varphi^2} + \frac{\partial^2 w_z}{\partial z^2},$$

$$w_r = w_\varphi = w_z = 0 \quad \text{at} \quad r = \xi, \quad r = 1 + \xi,$$
$$a = \mathrm{Re}\,(u_0 - c) + w_z.$$

It is necessary to seek a nonzero solution possessing a specified period $2\pi/\alpha$ along the z axis, where α is the wave number. The solution is sought in the form of series

$$\mathrm{Re} = \mathrm{Re}_* + \varepsilon^2, \quad w_r = \sum_{k=1}^{\infty} \varepsilon^k w_{rk}, \tag{2}$$
$$w_\varphi = \sum_{k=1}^{\infty} \varepsilon^k w_{\varphi k}, \quad p = \sum_{k=1}^{\infty} \varepsilon^k p_k,$$
$$\mathrm{Re}\, c = \mathrm{Re}_* \sum_{k=0}^{\infty} \varepsilon^k c_k, \quad w_z = \sum_{k=1}^{\infty} \varepsilon^k w_{zk},$$

where Re_* is the critical value of the Reynolds number according to the linear theory. Substituting (2) into (1) and equating the coefficients of identical powers of ε, we obtain an infinite sequence of systems of equations for determination of w_{rk}, $w_{\varphi k}$, w_{zk}, p_k, and c_k.

For k = 1 the system of equations is homogeneous, and its solution is of the form

$$\mathbf{w}_1 = \gamma\left(\mathbf{w}_0 e^{i(\alpha z + m\varphi)} + \overline{\mathbf{w}}_0 e^{-i(\alpha z + m\varphi)}\right), \quad p_1 = \gamma\left(q_1 e^{i(\alpha z + m\varphi)} + \overline{q}_1 e^{-i(\alpha z + m\varphi)}\right),$$

where $\mathbf{w}_0(w_{0r}, w_{0\varphi}, w_{0z})$, q_1 is the solution of the linear problem [1] and m is the azimuthal wave number. For k = 2 the term in the expansion (2) for the velocity and pressure perturbation components has the form

$$w_{2r} = \gamma^2[u_r + u_{1r}e^{2i(\alpha z + m\varphi)} + \overline{u}_{1r}e^{-2i(\alpha z + m\varphi)}],$$
$$w_{2\varphi} = \gamma^2[u_\varphi + u_{1\varphi}e^{2i(\alpha z + m\varphi)} + \overline{u}_{1\varphi}e^{-2i(\alpha z + m\varphi)}],$$
$$w_{2z} = \gamma^2[u_z + u_{1z}e^{2i(\alpha z + m\varphi)} + \overline{u}_{1z}e^{-2i(\alpha z + m\varphi)}],$$
$$p_2 = \gamma^2[q + q_2 e^{2i(\alpha z + m\varphi)} + \overline{q}_2 e^{-2i(\alpha z + m\varphi)}].$$

The functions $u(u_r, u_\varphi, u_z)$, q satisfy the following system of equations:

$$\left(\frac{1}{r}(r u_r)'\right)' + \frac{2\,\mathrm{Re}\, v_0}{r} u_\varphi - \mathrm{Re}\, q' = f_1,$$
$$\left(\frac{1}{r}(r u_\varphi)'\right)' - \mathrm{Re}\left(\frac{v_0}{r} + v_0'\right) u_r = f_2,$$
$$u_z'' + \frac{u_z'}{r} - \mathrm{Re}\, u_0' u_r = f_3, \quad (r u_r)' = 0$$

with the boundary conditions $u_r = u_\varphi = u_z = 0$ at $r = \xi$, $r = 1 + \xi$, where

$$f_1 = i\alpha\left(\overline{w}_{0z} w_{0r} - w_{0z}\overline{w}_{0r}\right) + w_{0r}\overline{w}_{0r}' + \overline{w}_{0r} w_{0r}' + \frac{im}{r}\left(\overline{w}_{0\varphi} w_{0r} - w_{0\varphi}\overline{w}_{0r}\right);$$
$$f_2 = i\alpha\left(\overline{w}_{0z} w_{0\varphi} - w_{0z}\overline{w}_{0\varphi}\right) + w_{0r}\overline{w}_{0\varphi}' + \overline{w}_{0r} w_{0\varphi}' + \frac{1}{r}\left(w_{0\varphi}\overline{w}_{0r} + \overline{w}_{0\varphi} w_{0r}\right);$$
$$f_3 = \frac{im}{r}\left(\overline{w}_{0\varphi} w_{0z} - w_{0\varphi}\overline{w}_{0z}\right) + w_{0r}\overline{w}_{0z}' + \overline{w}_{0r} w_{0z}'.$$

For determination of the functions $u_1(u_{1r}, u_{1\varphi}, u_{1z})$, q_2 we have the system

$$\left(\frac{1}{r}(ru_{1r})'\right)' - nu_{1r} + \mu u_{1\varphi} - \operatorname{Re} q_2' = f_1,$$

$$\left(\frac{1}{r}(ru_{1\varphi})'\right)' - nu_{1\varphi} - \sigma u_{1r} - \frac{2im}{r}\operatorname{Re} q_2 = f_2,$$

$$u_{1z}'' + \frac{u_{1z}'}{r} - nu_{1z} - \operatorname{Re} u_0' u_{1r} - 2i\alpha \operatorname{Re} q_2 = f_3,$$

$$(ru_{1r})' + 2i\alpha r u_{1z} + 2imu_{1\varphi} = 0$$

with the boundary conditions $u_{1r} = u_{1\varphi} = u_{1z} = 0$ at $r = \xi$, $r = \xi + 1$, where

$$f_1 = i\left(\alpha w_{0z} w_{0r} + \frac{m}{r} w_{0\varphi} w_{0r}\right) + w_{0r} w_{0r}' - \frac{w_{0\varphi}^2}{r^2};$$

$$f_2 = i\left(\alpha w_{0z} w_{0\varphi} + \frac{m}{r} w_{0\varphi}^2\right) + \frac{w_{0\varphi} w_{0r}}{r} + w_{0r} w_{0\varphi}';$$

$$f_3 = i\left(\alpha w_{0z}^2 + \frac{m}{r} w_{0\varphi} w_{0z}\right) + w_{0r} w_{0z}';$$

$$n = 2\left[i\alpha \operatorname{Re}(u_0 - c) + \frac{imv_0}{r}\operatorname{Re} + \frac{2m^2}{r^2} + 2\alpha^2\right];$$

$$\mu = \frac{2\operatorname{Re} v_0}{r} - \frac{4im}{r^2}; \quad \sigma = \operatorname{Re}\left(\frac{v_0}{r} + v_0'\right) - \frac{4im}{r^2}.$$

The condition of solvability of the systems of equations for $k > 1$ is equality of the c_k to zero for odd values of k. It is possible to convince oneself of this fact, since the solvability condition has the form

$$\int_0^{2\pi/\alpha}\int_0^{2\pi/m}\int_\xi^{\xi+1} f_k(r)\,\theta(r)\,e^{-i(\alpha z + m\varphi)} r\,dr\,dz\,d\varphi = 0,$$

where $f_k(r)$ are the right-hand sides of the equations of the system. The values $\theta(\theta_r, \theta_\varphi, \theta_z)$ and p_c are the solutions of the conjugate system

$$\theta_r'' - \frac{\theta_r'}{r} - (rb + \operatorname{Re} u_0')\,\theta_r - \left[\operatorname{Re}\left(\frac{v_0}{r} + v_0'\right) - \frac{2im}{r^2}\right]\theta_\varphi + rp_c' = 0,$$

$$\theta_\varphi'' - \frac{\theta_\varphi'}{r} + \left(\frac{2\operatorname{Re} v_0}{r} - \frac{2im}{r^2}\right)\theta_r - rb\theta_\varphi - imp_c = 0,$$

$$\theta_z'' - \frac{\theta_z'}{r} + \frac{\theta_z}{r^2} - \frac{b}{r}\theta_r - i\alpha r p_c = 0, \quad \theta_r' - \frac{im}{r}\theta_\varphi - i\alpha\theta_z = 0,$$

$$b = \frac{1}{r}\left[i\alpha \operatorname{Re}(u_0 - c) + \frac{m^2}{r^2} + \alpha^2 + \frac{im\operatorname{Re}}{r} v_0\right],$$

$$\theta_r = \theta_\varphi = \theta_z = 0 \quad \text{at} \quad r = \xi, \quad r = \xi + 1.$$

It follows from the system of equations for $k = 3$ that $w_3(w_{3r}, w_{3\varphi}, w_{3z})$, and p_3 are the sum of two harmonics with wave numbers α, m and 3α, 3m. The solvability condition for the system of equations for the amplitude of the first harmonic is of the form

$$-c_2 \operatorname{Re} J_1 + \gamma^2 J_2 + f J_3 = 0, \tag{3}$$

$$J_1 = \int_\xi^{\xi+1} i\alpha\,(w_{0r}\theta_r + w_{0\varphi}\theta_\varphi + w_{0z}\theta_z)\,r\,dr,$$

$$J_2 = \int_\xi^{\xi+1}\Big\{\Big[i\alpha\,(2\bar{w}_{0z}u_{1r} + w_{0r}u_z - \bar{w}_{0r}u_{1z}) + \bar{w}_{0r}'u_{1r} + \bar{w}_{0r}u_{1r}' +$$

$$+ \frac{im}{r}(2w_{0\varphi}u_{1r} + w_{0r}u_\varphi - \bar{w}_{0r}u_{1\varphi}) - \frac{2}{r}(w_{0\varphi}u_\varphi + \bar{w}_{0\varphi}u_{1\varphi})\Big]\theta_r +$$

$$+ \Big[i\alpha\,(2\bar{w}_{0z}u_{1\varphi} + w_{0\varphi}u_z - \bar{w}_{0\varphi}u_{1z}) + w_{0r}u_\varphi' + \bar{w}_{0r}u_{1\varphi}' + \bar{w}_{0\varphi}'u_{1r} +$$

$$+ \frac{im}{r}(2\bar{w}_{0\varphi}u_{1\varphi} + w_{0\varphi}u_\varphi - \bar{w}_{0\varphi}u_{1\varphi}) + \frac{1}{r}(w_{0\varphi}u_{1r} + w_{0r}u_\varphi + \bar{w}_{0r}u_{1\varphi})\Big]\theta_\varphi +$$

$$+\left[i\alpha\left(2\bar{w}_{0z}u_{1z}+w_{0z}u_z-\bar{w}_{0z}u_{1z}\right)+w_{0r}u_z'+\bar{w}_{0r}u_{1z}'+\bar{w}_{0z}'u_{1r}+\right.$$

$$\left.\left.+\frac{im}{r}\left(2\bar{w}_{0\varphi}u_{1z}+w_{0z}u_\varphi-\bar{w}_{0z}u_{1\varphi}\right)\right]\theta_z\right\}rdr,$$

$$J_3=\int\limits_{\xi}^{\xi+1}\left\{\left[i\alpha\left(u_0+\frac{m}{\alpha r}v_0\right)w_{0r}-\frac{2v_0}{r}w_{0\varphi}+q_1'\right]\theta_r+\left[i\alpha\left(u_0+\frac{m}{\alpha r}v_0\right)w_{0\varphi}+\right.\right.$$

$$\left.\left.+\left(\frac{v_0}{r}+v_0'\right)w_{0r}+\frac{im}{r}q_1\right]\theta_\varphi+\left[i\alpha\left(u_0+\frac{m}{\alpha r}v_0\right)w_{0z}+u_0'w_{0r}+i\alpha q_1\right]\theta_z\right\}rdr.$$

The sign of f is selected from the condition $\gamma^2 > 0$. If the constant γ^2 determined from (3) turns out to be positive, then the existence of a supercritical self-oscillating mode follows from the results [5]. In the opposite case only subcritical self-oscillations exist.

Let us dwell on the procedure of specific calculations. For fixed Re and α belonging to the neutral curve, the phase velocity of the perturbations c_0 is found as the solution of the linear problem for the eigenvalues by the method of differential elimination with splicing at the critical point [6]. For known Re and c_0 the eigenfunction $\mathbf{w}_0(w_{0r}, w_{0\varphi}, w_{0z})$, q_1 is found by backward elimination from the already-known elimination relations. Normalization is performed at the critical point. The components θ_r, θ_φ, θ_z of the conjugate system of equations are calculated in an analogous way. In connection with the solution of a nonhomogeneous system of equations, the method of differential elimination is also used, and the elimination relations are of the form

$$\mathbf{Z}=A\mathbf{X}+\mathbf{B},$$

where A is the elimination matrix, **Z** and **X** are vectors formulated from the unknown functions and their derivatives, and the vector **B** is introduced due to the inhomogeneity of the equations.

The systems of ordinary differential equations for the elimination coefficients with the appropriate initial conditions on one of the boundaries are integrated by the Runge–Kutta method with automatic choice of step length. Integration from the second boundary is performed similarly. The initial values for the reverse elimination are found from the elimination relations at the splicing point. The integrals J_1, J_2, and J_3 are calculated by Simpson's method.

As follows from [1], if the axial Reynolds numbers are small, convective instability develops initially for a specified rotational velocity of the inner cylinder, and then viscous instability arises as β increases. Such a pattern is characteristic of different gaps between the cylinders.

In the limit of Re = 0 a stable self-oscillating mode branches off weakly from the convective-type neutral curve for supercritical values of the Taylor numbers [3].

The numerical calculations have shown that weak excitation of a Taylor instability is maintained for Re $\neq$ 0. The stability diagram of a spiral flow in a narrow gap (ξ = 50) in the Re, β plane is illustrated in Fig. 1. Curve 1 specifies the variation of $Re_*(\beta)$ for convective instability, and the $Re_*(\beta)$ of the nubs of the viscous-type neutral curves varies along curve 1'. The arrows indicate where self-oscillating modes exist. In the range of variation of Re in which convective instability precedes viscous instability, stable self-oscillations branch off from the viscous-type neutral curves in the case of weak supercriticality. The results of the calculation agree with the data of [7], where the nature of the branching of secondary self-oscillating modes is exhibited for Re $\leqslant$ 40 and ξ = 50.

For several values of β (β = 0.285, 0.36, and 0.16) an analysis was made of the nature of branching along the viscous-type (Fig. 2, curves 1, 2, β = 0.36, 0.285) and Taylor-type (curves 3, 4, β = 0.285, 0.16) neutral curves. When β > 0.1, a self-oscillating mode branches off inside the neutral curves for $\alpha = \alpha_*$, as well as for all α on the upper and lower branches of neutral curves of the viscous and Taylor type, which corresponds to weak excitation of secondary instability. If $\beta \sim 0.09$, a wave number α appears on the upper branch of the viscous-type neutral curve for which a change occurs in the nature of the branching, i.e., a self-oscillating mode exists for Re < Re_*, but no self-oscillations arise on the nub of the neutral curve in the case of supercritical values of the Reynolds number. As the relative rotational velocity of the inner cylinder decreases further, the point of change in the nature of the branching is shifted from the upper branch of the neutral curve to the lower

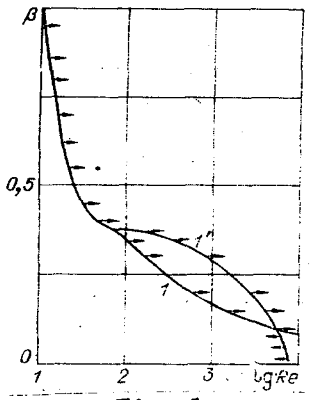

Fig. 1

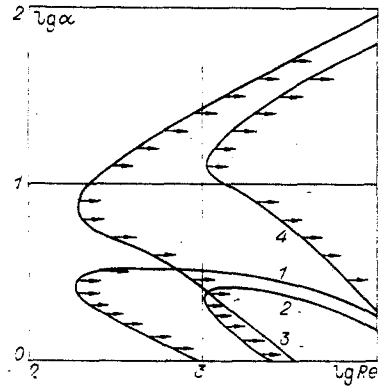

Fig. 2

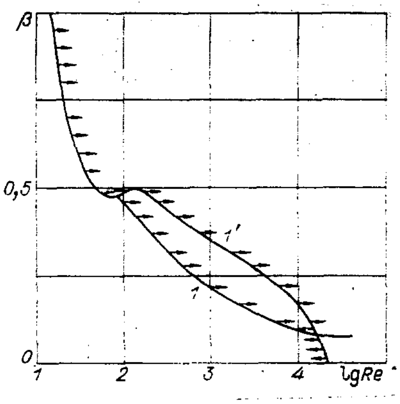

Fig. 3

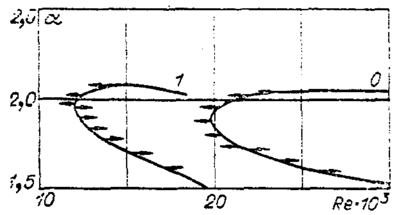

Fig. 4

one, and for $\beta = 0$ this point is characterized by the parameters $\alpha = 1.8$ and Re = 9261, which agrees with the data of [4, 8].

When $\beta \sim 0.08$, the viscous instability becomes the most dangerous. It turns out that for such a flow mode in which convective instability develops against the background of a developed viscous instability, the nature of the branching of the secondary self-oscillations at the nub of the viscous-type neutral curves changes, which has a fundamental meaning. If the self-oscillating mode for $\alpha = \alpha_*$ branches off in the direction of smaller Reynolds numbers, the excitation is of an entirely rigorous nature.

The stability diagram in the Re, β plane of the spiral flow between coaxial cylinders when the radius of the inner cylinder is equal to one-half that of the outer cylinder ($\xi = 1$, $m = 0$) is illustrated in Fig. 3. Places where self-oscillating modes exist are indicated by arrows. The most dangerous perturbations are considered. Curve 1 corresponds to the variation of $Re_*(\beta)$ for the nubs of the convective-type neutral curves. The critical value of the Reynolds numbers varies along curve 1′ with the variation in the twist of the inner cylinder for viscous-type neutral curves.

For small axial Reynolds numbers Re and in the range of variation of Re in which convective instability precedes the viscous, branching off of secondary self-oscillating modes occurs within the neutral curves of the viscous and Taylor type, i.e., weak excitation, as in the two-dimensional case ($\xi = 50$), is characteristic both for Taylor and viscous instability. If $\beta \leqslant 0.09$ and $Re \geqslant 12{,}000$, the flow mode is altered in the sense that viscous instability develops at first, and then Taylor vortices appear against its background. The nature of the excitation of secondary modes for viscous-type neutral curves is altered similarly to the two-dimensional case. The change in the nature of the branching is characterized by the parameters $\beta \sim 0.09$, Re = 12,100, and $\alpha = 1.96$.

For $\alpha = \alpha_*$ and all α on the upper branch of the viscous-type neutral curve the self-oscillating solution branches off into the region of stability of the original laminar flow. For $\alpha < \alpha_*$ and for all α on the lower branch a stable self-oscillating solution exists inside the loop of the neutral curve. A decrease in β results in the transition of the point of change of the branching nature to the lower branch of the neutral curve, and for $\beta = 0$ this point is characterized by the following parameters: $\alpha \sim 1.72$, Re = 21,600. The viscous-type neutral curves for $\beta = 0$ and 0.09 are illustrated in Fig. 4 (curves 0 and 1).

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AN OPTICAL METHOD FOR INVESTIGATING THE MICROSTRUCTURE OF TURBULENT FLOW

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In a number of cases the study of the structure of small-scale high-intensity turbulence shows a deviation from Taylor's hypothesis which is manifested in incompatible values of time and spatial correlations [1]. In view of this the spatial structure of the flow cannot be estimated from a measurement of the autocorrelation of processes recorded by point transducer such as the various types of anemometers widely used to study turbulence. In this case it becomes important to use instruments with an analyzing volume larger than the microscale of the phenomenon under study and methods enabling one to record the state of a chosen volume at a certain instant with the simultaneous visualization of the flow. In particular, this metho was used in [2] for the quantitative determinations of certain characteristics of turbulence by introducing tracer particles into the medium and recording their distribution through the volume photographically. In many cases shadow optical instruments [3, 4] are used for the visualization of optical inhomogeneities in transparent media. Their use has a number of advantages over visualization methods employing finely dispersed materials or dyestuffs, since they avoid inertial effects which always accompany the introduction of tracer particles

We examine certain problems of the statistical analysis of flow domains whose dimensions are determined by the diameter of the light beam of the shadow instrument. Since the recorders used in investigating the microstructure of these domains are at best two-dimensional, wherea the field parameter being measured is a multidimensional quantity, it must be established how this affects the nature of the statistical field data being measured. One of the fundamental characteristics of a field is its wave-number power spectrum $g(\overline{k})$, in certain cases called th Wiener spectrum, and by analogy with one-dimensional processes having the meaning of the average variance of the spectral components with spatial frequencies in a small interval around $\overline{k}$ divided by the size of this interval. The use of measuring devices having a dimensionality smaller than that of the field being measured is equivalent to the effect of spectral windows which change the characteristics of the estimate of the spectrum. The dimensionality of a spectral window corresponds to that of the recording device [5]. In other words, the width of a spectral window along one of the frequency coordinates is inversely proportional to the sample length with respect to the corresponding spatial or time coor-

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